Announcements

1) Exam 3 also covers 3.2
(Mean Value Theorem)

2) More practice problems on CTools,
also practice exam

3) In-class review tomorrow,
Amanda's review at 4:30

Example 1: $\int \sqrt{9+x^4}\; x^{11}\, dx$

$u = 9 + x^4$

$du = 4x^3\, dx$

Looks like it won't help,

but

$\int \sqrt{9+x^4}\; x^{11}\, dx$

$= \int \sqrt{9+x^4} \cdot x^4 \cdot x^4 \cdot x^3\, dx$

If $u = 9 + x^4$, $x^4 = u - 9$

and $du = 4x^3 dx$, so $\frac{du}{4} = x^3 dx$

Substitute in.

$$\int \sqrt{9+x^4} \cdot x^{11} dx$$

$$= \int \sqrt{9+x^4} \cdot x^4 \cdot x^4 \cdot \boxed{x^3 dx}$$

$$= \int \sqrt{u} \cdot (u-9)(u-9) \cdot \frac{du}{4}$$

$$= \frac{1}{4} \int \sqrt{u} \left(u^2 - 18u + 81\right) du$$

$$= \frac{1}{4} \int u^{5/2} - 18u^{3/2} + 81u^{1/2} du$$

$$= \frac{1}{4} \int u^{5/2} - 18u^{3/2} + 81u^{1/2} \, du$$

$$= \frac{1}{4}\left(\frac{2}{7} u^{7/2} - 18 \cdot \frac{2}{5} \cdot u^{5/2} + 81 \cdot \frac{2}{3} u^{3/2} + C \right)$$

$$\boxed{= \frac{1}{4}\left(\frac{2(9+x^4)^{7/2}}{7} - \frac{36}{5}(9+x^4)^{5/2} + 54(9+x^4)^{3/2} \right) + C}$$

Example 2 , $\displaystyle\int_3^8 \frac{x}{\sqrt{x+1}}\,dx$

$u = x+1$

$du = dx$

$x = u-1$

when $x=3$, $u=4$ and

when $x=8$, $u=9$. We get

$$\int_3^8 \frac{x}{\sqrt{x+1}}\,dx$$

$$\frac{u}{\sqrt{u}} = \frac{u^1}{u^{1/2}} = u^{1-1/2} = u^{1/2}$$

$$= \int_4^9 \frac{u-1}{\sqrt{u}}\,du$$

$$= \int_4^9 \boxed{\frac{u}{\sqrt{u}}} - \frac{1}{\sqrt{u}}\,du$$

$$= \int_4^9 u^{1/2} - u^{-1/2}\,du$$

$$\int_4^9 u^{1/2} - u^{-1/2}\, du$$

$$= \left(\frac{2u^{3/2}}{3} - 2u^{1/2} \right) \Big|_4^9$$

$$= \left(\frac{2(9^{3/2})}{3} - 2\sqrt{9} \right) - \left(\frac{2}{3}(4^{3/2}) - 2\sqrt{4} \right)$$

$$= \left(\frac{54}{3} - 6 \right) - \left(\frac{16}{3} - 4 \right)$$

$$= \boxed{\frac{38}{3} - 2}$$

Area Between Curves

(Section 5.1)

Recall area: Area between the x-axis and the graph of $y = f(x)$ from $x = a$ to $x = b$ is

$$A = \int_a^b |f(x)|\, dx$$

Definition (area between curves)

The area between $y=f(x)$ and $y=g(x)$ from $x=a$ to $x=b$ is

$$A = \int_a^b |f(x)-g(x)|\,dx$$

Warnings: 1) There is always some algebra to do, since

$$|f(x)-g(x)| = \begin{cases} f(x)-g(x) & \text{if } f \geq g \\ g(x)-f(x) & \text{if } g \geq f \end{cases}$$

You need to find where $f(x)-g(x)=0$!

2) $\int |f(x)-g(x)|\,dx$

$\neq \int |f(x)|-|g(x)|\,dx$

in general.

Example 3: $f(x) = x^3$, $g(x) = x^2$

from $x = \frac{1}{3}$ to $x = 2$.

$$A = \int_{\frac{1}{3}}^{2} |x^3 - x^2| \, dx$$

Find where $x^3 - x^2 = 0$.

$$0 = x^3 - x^2 = x^2(x-1)$$

$x = 0$ or $x = 1$

0 is not in [1/3, 2], so ignore it. However, one is in [1/3, 2]. So

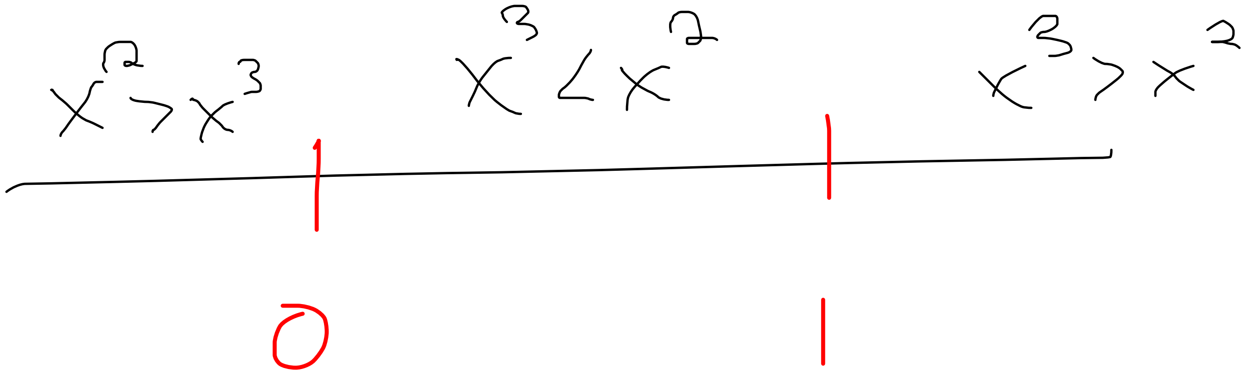
break up integral

$$\int_{1/3}^{2} |x^3 - x^2|\,dx = \int_{1/3}^{1} |x^3 - x^2|\,dx + \int_{1}^{2} |x^3 - x^2|\,dx$$

$$= \int_{1/3}^{1} x^2 - x^3\,dx + \int_{1}^{2} x^3 - x^2\,dx$$

$$= \left(\frac{x^3}{3} - \frac{x^4}{4}\right)\Bigg|_{1/3}^{1} + \left(\frac{x^4}{4} - \frac{x^3}{3}\right)\Bigg|_{1}^{2}$$

$$= \boxed{\frac{1}{12} - \left(\frac{1}{81} - \frac{1}{324}\right) + \left(4 - \frac{8}{3} + \frac{1}{12}\right)}$$


$x^2 > x^3$
$x^3 < x^2$
$x^3 > x^2$
0
1

Example 4: Between $\sin(2x)$ and $\cos(x)$ from $x=0$ to $x=\frac{7\pi}{6}$

$$A = \int_0^{7\pi/6} \left|\sin(2x) - \cos(x)\right| dx$$

Find where $\sin(2x) - \cos(x) = 0$.

Use trig identity

$$\sin(2x) = 2\sin(x)\cos(x)$$

So

$$2\sin(x)\cos(x) - \cos(x) = 0$$

$$\cos(x)\left(2\sin(x) - 1\right) = 0$$

So either $\cos(x) = 0$

or $2\sin(x) - 1 = 0$

(same as $\sin(x) = \frac{1}{2}$)

In between 0 and $\frac{7\pi}{6}$, the solution(s) to $\cos(x) = 0$ is $x = \frac{\pi}{2}$

and to $\sin(x) = \frac{1}{2}$ $x = \frac{\pi}{6}, \frac{5\pi}{6}$

So

$$A = \int_0^{7\pi/6} |\sin(2x) - \cos(x)|\, dx$$

$$= \int_0^{\pi/6} |\sin(2x) - \cos(x)|\, dx$$

$$+ \int_{\pi/6}^{\pi/2} |\sin(2x) - \cos(x)|\, dx$$

$$+ \int_{\pi/2}^{5\pi/6} |\sin(2x) - \cos(x)|\, dx$$

$$+ \int_{5\pi/6}^{7\pi/6} |\sin(2x) - \cos(x)|\, dx$$

$$= \int_0^{\pi/6} -\sin(2x) + \cos(x)\,dx$$

$$+ \int_{\frac{\pi}{6}}^{\pi/2} \sin(2x) - \cos(x)\,dx$$

$$+ \int_{\pi/2}^{5\pi/6} -\sin(2x) + \cos(x)\,dx$$

$$+ \int_{5\pi/6}^{7\pi/6} \sin(2x) - \cos(x)\,dx$$

Do the integration!